HYDRO DYNAMICS

FLUID FLOW MEASUREMENT

STEADY FLOW

Fluid Flow Measurement

Introduction

There are numerous number of devices used to measure the flow of fluids. In any of these devices, the Bernoulli's Energy Theorem is greatly utilized and additional knowledge of the characteristics and coefficients of each device is important. In the absence of reliable values and coefficients, a device should be calibrated for the expected operating conditions.

Fluid Flow Measurement

Device Coefficients

Coefficient of Discharge, $\boldsymbol{C}$ *or* $\boldsymbol{C_d}$

The coefficient of discharge is the ratio of the actual discharge through the device to the ideal or theoretical discharge which would occur without losses. This may be expressed as:

$$\boldsymbol{C}\ or\ \boldsymbol{C_d} = \frac{Actual\ discharge}{Theoretical\ discharge} = \frac{Q}{Q_T}$$

The actual discharge may be accomplished by series of observation, usually by measuring the total amount of fluid passing through the device for a known period. The theoretical value can be accomplished using the Bernoulli's Energy Theorem neglecting losses.

Fluid Flow Measurement

Device Coefficients

Coefficient of Velocity, C_v

The coefficient of velocity is the ratio of the actual mean velocity to the ideal or theoretical mean velocity which would occur without any losses.

$$C_v = \frac{Actual\ velocity}{Theoretical\ velocity} = \frac{v}{v_T}$$

Coefficient of Contraction, C_c

The coefficient of contraction is the ratio of the actual area of the contracted section of the stream or jet to the area of the opening through which the fluid flows.

$$C_c = \frac{Area\ of\ stream\ or\ jet}{Area\ of\ opening} = \frac{a}{A}$$

Fluid Flow Measurement

Device Coefficients

Relationship between the three coefficients

$Actual\ discharge, Q = C\ x\ Q_T$

Also, $Actual\ discharge, Q = Actual\ area\ (a) x\ Actual\ velocity\ (v)$

$Q = C_c A\ x\ C_v v_T$

$Q = C_c C_v A v_T$

But $A v_T = Q_T$

$Q = C_c C_v Q_T$

$$\boldsymbol{C}\ or\ \boldsymbol{C_d} = \boldsymbol{C_c}\ \boldsymbol{x}\ \boldsymbol{C_v}$$

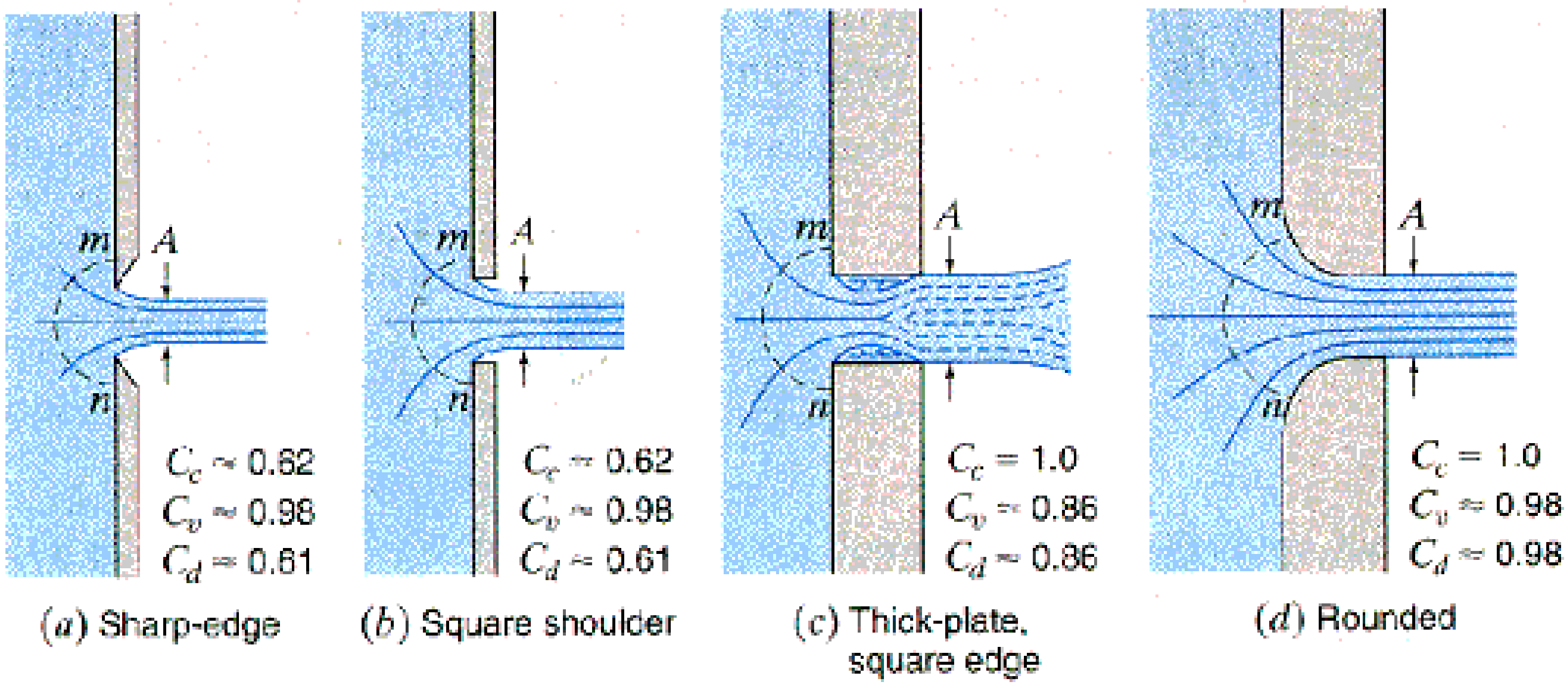
The coefficient of discharge varies with Reynolds Number. It is not constant for a given device.

Fluid Flow Measurement

Device Coefficients

Relationship between the three coefficients

$$C \text{ or } C_d = C_c \times C_v$$



Fluid Flow Measurement

Head Loss

The head loss through Venturi meters, orifices, tubes, and nozzles may be expressed as:

The ideal energy equation between ***1*** and ***2*** is:

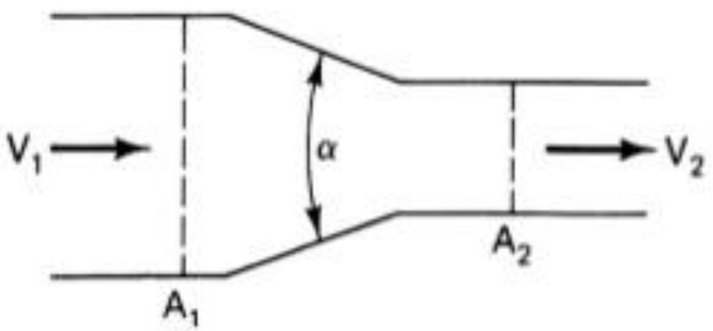
$$E_1 = E_2$$

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$A_1 v_1 = A_2 v_2$$

$$v_1 = \frac{A_2}{A_1} v_2$$

$$\frac{v_1^2}{2g} = \left(\frac{A_2}{A_1}\right)^2 \frac{v_2^2}{2g}$$



Fluid Flow Measurement

Head Loss

$$\left(\frac{A_2}{A_1}\right)^2 \frac{v_2^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$\left[1 - \left(\frac{A_2}{A_1}\right)^2\right] \frac{v_2^2}{2g} = \left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right)$$

$$(v_2)_{theoretical} = \frac{1}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{2g\left[\left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right)\right]} \qquad \textbf{(a)}$$

Considering head loss between ***1*** and ***2***:

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - HL = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$(v_2)_{actual} = \frac{1}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{2g\left[\left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right) - HL\right]} \qquad \textbf{(b)}$$

Fluid Flow Measurement

Head Loss

Since $v_{actual} = C_v v_{theoretical} = v$

$$v = C_v \frac{1}{\sqrt{1-(A_2/A_1)^2}} \sqrt{2g\left[\left(\frac{p_1}{\gamma}+z_1\right)-\left(\frac{p_2}{\gamma}+z_2\right)\right]}$$

Squaring both sides and arranging terms:

$$\left[1-\left(\frac{A_2}{A_1}\right)^2\right]\frac{v^2}{2g}\left(\frac{1}{C_v^2}\right) = \left(\frac{p_1}{\gamma}+z_1\right)-\left(\frac{p_2}{\gamma}+z_2\right)$$

From ***(b)***

$$\left(\frac{p_1}{\gamma}+z_1\right)-\left(\frac{p_2}{\gamma}+z_2\right) = \left[1-\left(\frac{A_2}{A_1}\right)^2\right]\frac{v^2}{2g}+HL$$

$$\left[1-\left(\frac{A_2}{A_1}\right)^2\right]\frac{v^2}{2g}\left(\frac{1}{C_v^2}\right) = \left[1-\left(\frac{A_2}{A_1}\right)^2\right]\frac{v^2}{2g}+HL$$

Fluid Flow Measurement

Head Loss

$$HL = \left[1 - \left(\frac{A_2}{A_1}\right)^2\right]\frac{v^2}{2g}\left(\frac{1}{C_v^2}\right) - \left[1 - \left(\frac{A_2}{A_1}\right)^2\right]\frac{v^2}{2g}$$

$$HL = \left(\frac{1}{C_v^2} - 1\right)\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]\frac{v^2}{2g} \qquad \textbf{(c)}$$

If the orifice or nozzle takes off directly from a tank where A_1 is very much greater than A_2, then the velocity of approach is negligible and Eqn. ***(c)*** reduces to:

$$HL = \left(\frac{1}{C_v^2} - 1\right)\frac{v^2}{2g} \qquad \textbf{(d)}$$

Note: *v = actual velocity*

Fluid Flow Measurement

Orifice

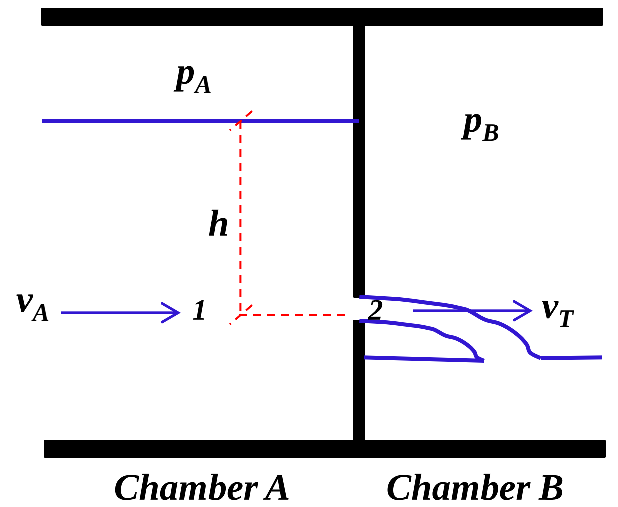
An *orifice* is an opening (usually circular) with a closed perimeter through which fluid flows. It is used primarily to measure or to control the flow of fluid. The upstream face of the orifice may be rounded or sharp. An orifice with prolonged side, such as a piece of pipe, having a length of two or three times its diameter, is called a *short tube*. Longer tubes such as culverts under embankments are usually treated as orifice although they may be also treated as *short pipes*.

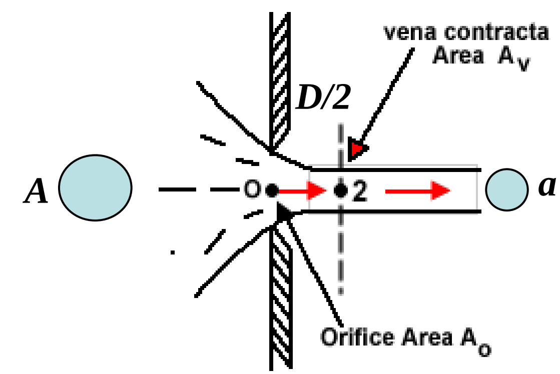
According to shape, orifice may be *circular*, *square*, or *rectangular* in cross-section. The *circular sharp-crested orifice* is most widely used because of the simplicity of its design and construction.

Fluid Flow Measurement

Orifice

The figure below shows a general case of fluid flow through an orifice. Let p_A and p_B be the air pressures in the chambers *A* and *B*, respectively and v_A be the velocity of the stream normal to the plane of the orifice (*velocity of approach*). Consider two points *1* and *2* such that $v_1 = v_A$ and $v_2 = v_T$ and writing the energy equation between these two points neglecting losses:





$$E_1 = E_2$$

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

Note: Vena contracta is the section on the jet where the contraction ceases

Fluid Flow Measurement

Orifice

$$E_1 = E_2$$

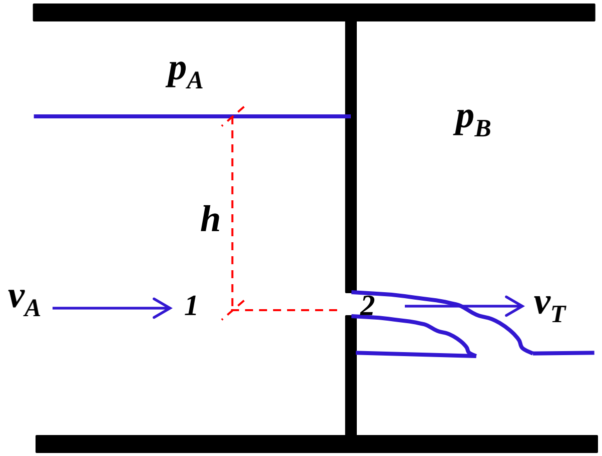
$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

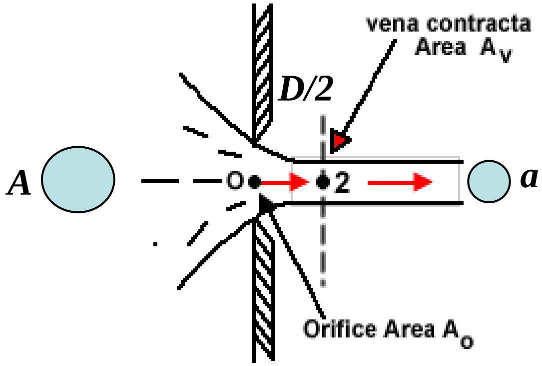
$$\frac{v_A^2}{2g} + \frac{p_A + \gamma h}{\gamma} + 0 = \frac{v^2}{2g} + \frac{p_B}{\gamma} + 0$$

$$\frac{v_A^2}{2g} + \frac{p_A}{\gamma} + h = \frac{v_T^2}{2g} + \frac{p_B}{\gamma}$$

$$\frac{v_T^2}{2g} = h + \frac{p_A}{\gamma} - \frac{p_B}{\gamma} + \frac{v_A^2}{2g}$$

$$v_T = \sqrt{2g\left[h + \frac{v_A^2}{2g} + \left(\frac{p_A}{\gamma} - \frac{p_B}{\gamma}\right)\right]}$$





Fluid Flow Measurement

Orifice

Theoretical velocity,

$$v_T = \sqrt{2g\left[h + \frac{v_A^2}{2g} + \left(\frac{p_A}{\gamma} - \frac{p_B}{\gamma}\right)\right]}$$

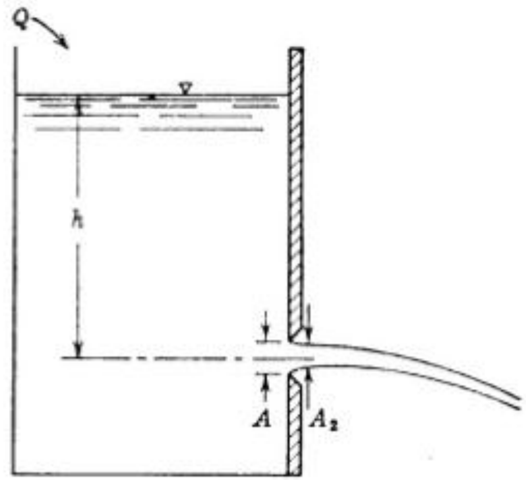
If the surface of the liquid in chamber ***A*** and the jet in chamber ***B*** are each exposed to the atmosphere, and also the cross-sectional area of the reservoir or channel leading to the orifice is large in comparison with the area of the orifice the velocity of approach becomes negligible, and theoretical velocity is:

$$v_T = \sqrt{2gH}$$

H = total head producing flow (m)

H = Head upstream – Head downstream

Free discharge orifice



$$v_T = \sqrt{2gH}$$

$$v = C_v\sqrt{2gH}$$

$$Q_T = A\sqrt{2gH}$$

$$Q = CA\sqrt{2gH}$$

$$H = h + \frac{v_A^2}{2g} + \frac{p_A}{\gamma} - \frac{p_B}{\gamma}$$

Fluid Flow Measurement

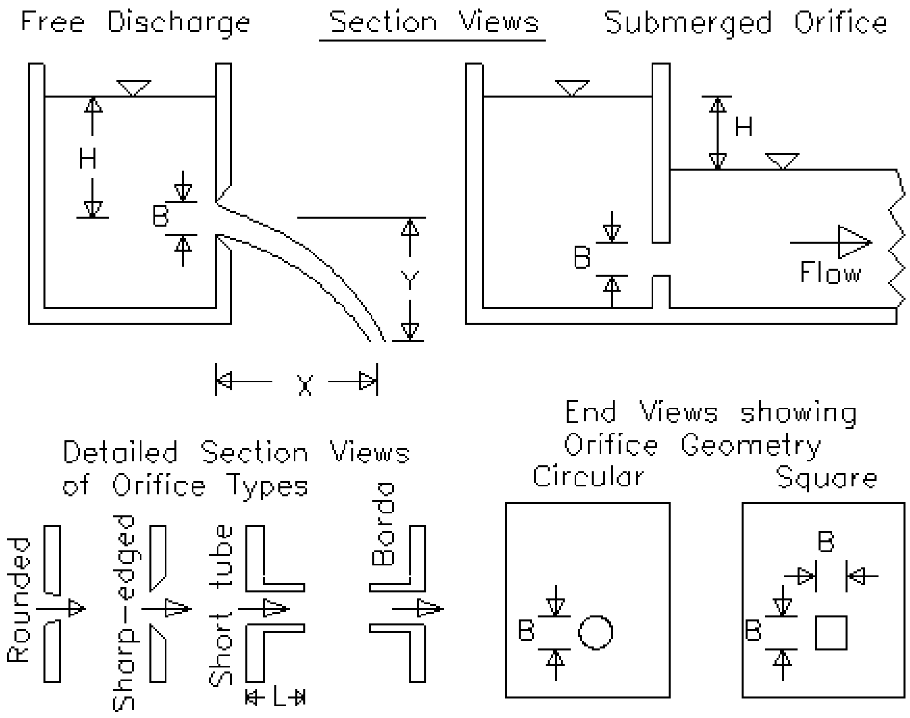
Orifice

Torricelli's Theorem

The theoretical velocity of discharge from an orifice is the velocity acquired by a body falling freely in a vacuum through a height equal to the total head on the orifice.

$$v_T = \sqrt{2gH}$$

$$H = h + \frac{v_A^2}{2g} + \frac{p_A}{\gamma} - \frac{p_B}{\gamma}$$

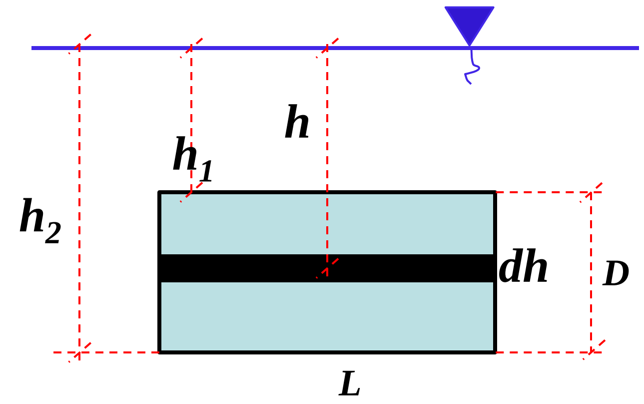


Fluid Flow Measurement

Orifice

Orifices under low heads

When the head on a vertical orifice is small in comparison with the height of the orifice, there is an appreciable difference between the discharges using the previous analysis.



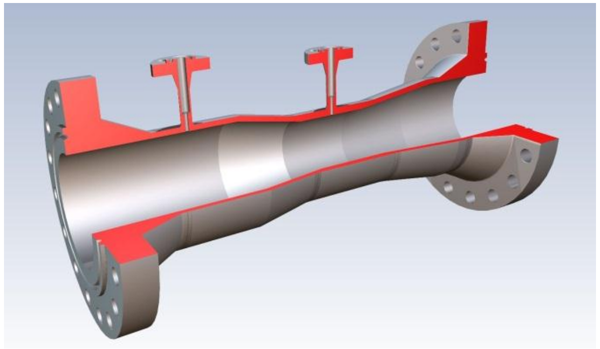
Consider the rectangular section of length $\boldsymbol{L}$ and height $\boldsymbol{D}$ as shown in the figure with both the surface and the jet subject to atmospheric pressure. The theoretical discharge through an elementary strip of length $\boldsymbol{L}$ and height $\boldsymbol{dh}$ is:

$$dQ_T = (Ldh)\sqrt{2gh} \rightarrow Q_T = \frac{2}{3}\sqrt{2g}\,L\left(h_2^{3/2} - h_1^{3/2}\right) \rightarrow \boldsymbol{Q = \frac{2}{3}C\sqrt{2g}\,L\left(h_2^{3/2} - h_1^{3/2}\right)}$$

Fluid Flow Measurement

Venturi Meter

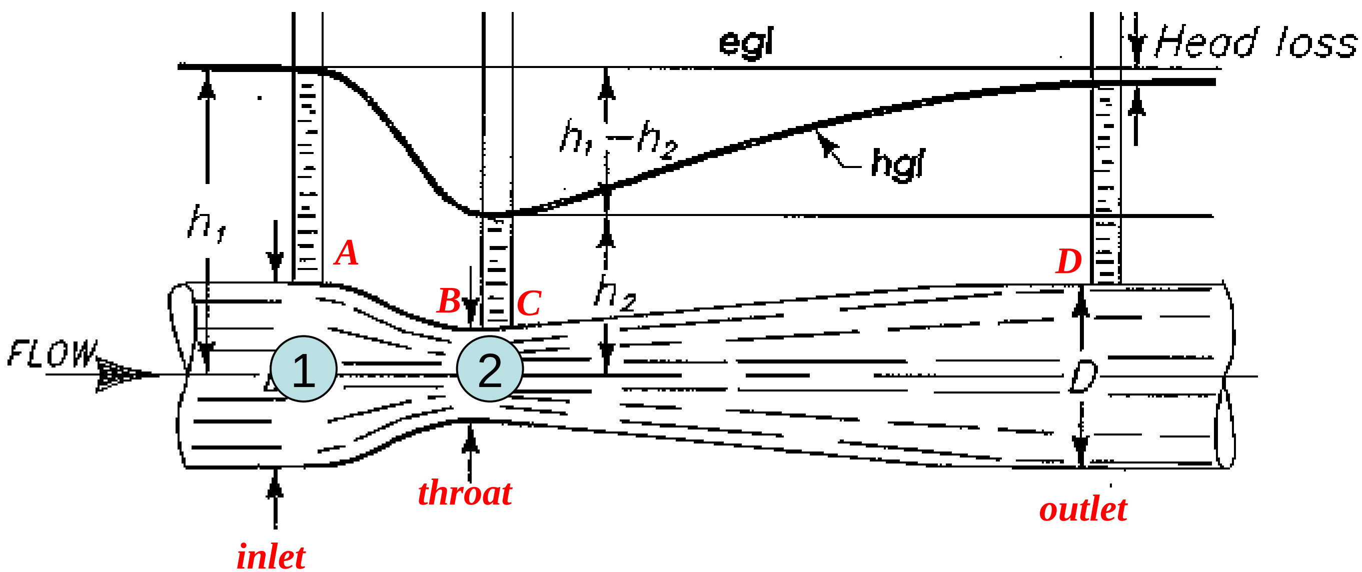
Venturi meter is an instrument used in measuring the discharge through pipes.



Fluid Flow Measurement

Venturi Meter

It consist of a converging tube ***AB*** which is connected to the main pipe at the inlet ***A***, and ending in a cylindrical section ***BC*** called the *throat*, and a diverging section ***CD*** which is connected again to the main pipe at the outlet ***D***. The angle of divergence is kept small to reduce the head loss cause by turbulence as the velocity is reduced.



Fluid Flow Measurement

Venturi Meter

Consider two points in the system, ***1*** at the base of the inlet and ***2*** at the throat, and writing the energy equation between these two points neglecting head loss:

$$E_1 = E_2$$

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

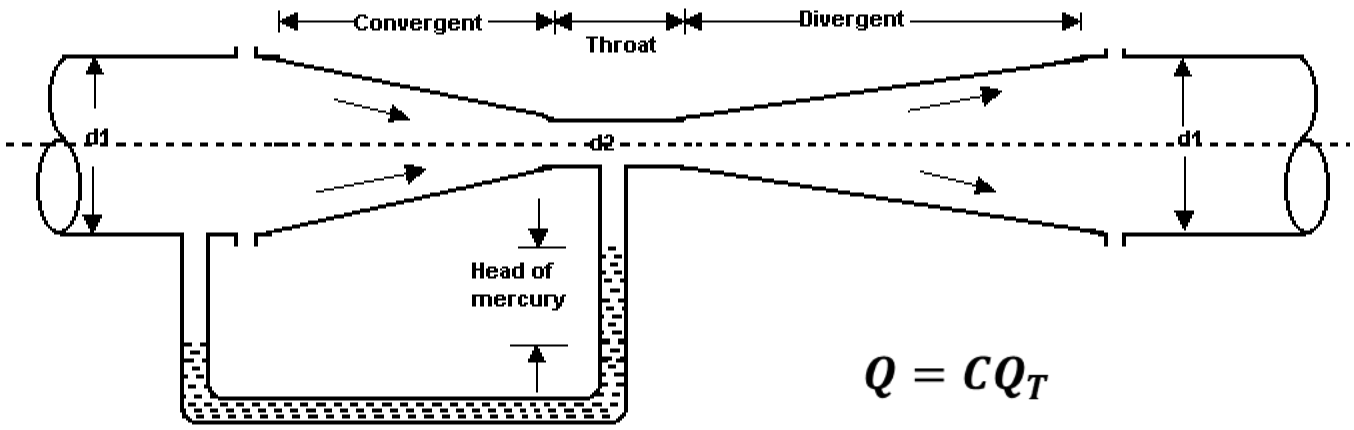
$$\boldsymbol{\frac{v_2^2}{2g} - \frac{v_1^2}{2g} = \left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right)}$$

The left side of the equation is the *kinetic energy* which shows an increase in value, while the left side of the equation is the *potential energy* which shows a decrease in value. Therefore, neglecting head loss, the increase in kinetic energy is equal to the decrease in potential energy. This statement is known as the *Venturi Principle*.

Fluid Flow Measurement

Venturi Meter

The difference in pressure between the inlet and the throat is commonly measured by means of a differential manometer connecting the inlet and throat.



$$Q = CQ_T$$

If the elevations and the difference in pressure between ***1*** and ***2*** are known, the discharge (*theoretical or ideal*) can be solved.

Fluid Flow Measurement

Nozzle

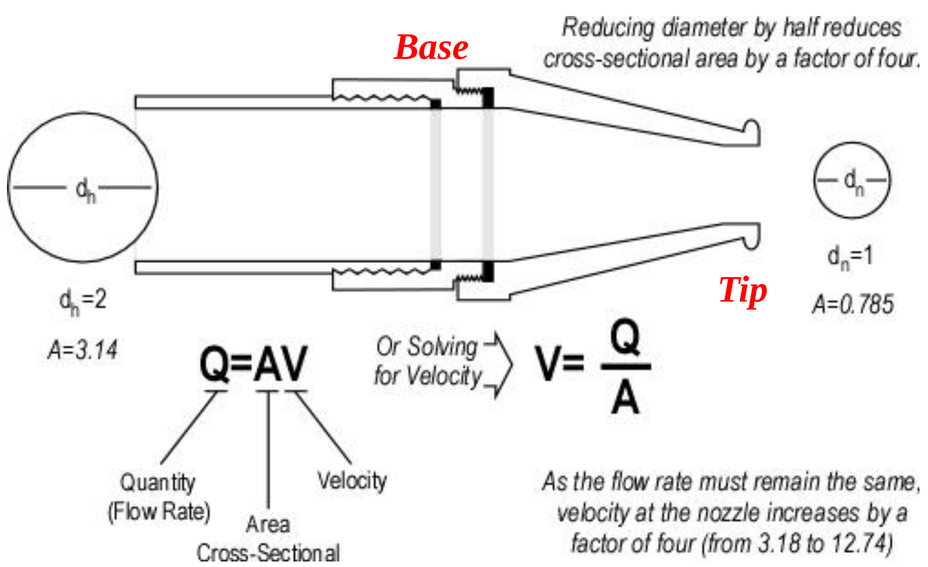
A *nozzle* is a converging tube installed at the end of a pipe or hose for the purpose of increasing the velocity of the issuing jet.

$$Q = CA_n\sqrt{2gH}$$

where,

H = total head at base of nozzle

A_n = Area at the nozzle tip



The head loss through a nozzle is given by Eqn. ***(c)*** *–* ***(d)***.

Fluid Flow Measurement

Pitot Tube

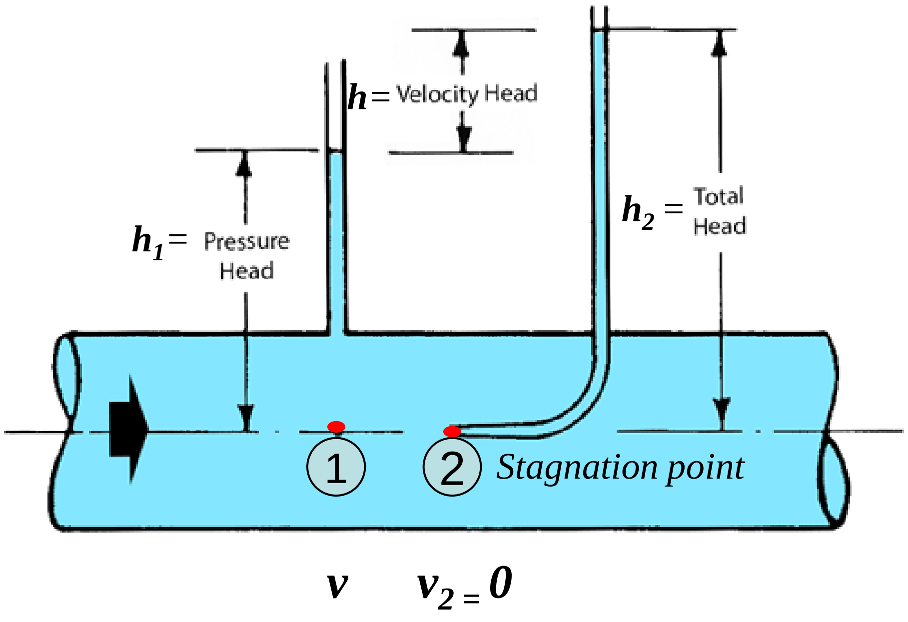
Named after the French physicist and engineer Henri Pitot, *Pitot tube* is a bent (*L*-shaped or *U*-shaped) tube with both ends open and is used to measure the velocity of fluid flow.

When the tube is placed in a moving stream with open end oriented into the direction of flow, the liquid enters the opening at point ***2*** until the surface in the tube rises a distance of ***h*** above the stream surface. An equilibrium condition is then established, and the quantity of liquid in the tube remains unchanged as the flow remains steady.

Fluid Flow Measurement

Pitot Tube

Point ***2*** at the face of the tube facing the stream is called the *stagnation point.*



Kinetic energy is transformed to potential energy.

Consider a particle at point ***1*** to move with a velocity of ***v***. As the particle approaches point ***2***, its velocity is gradually retarded to ***0*** at point ***2***. Writing the energy equation between ***1*** and ***2*** neglecting losses:

$$E_1 = E_2$$

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + \cancel{z_1}^{\,0} = \frac{\cancel{v_2^2}^{\,0}}{2g} + \frac{p_2}{\gamma} + \cancel{z_2}^{\,0}$$

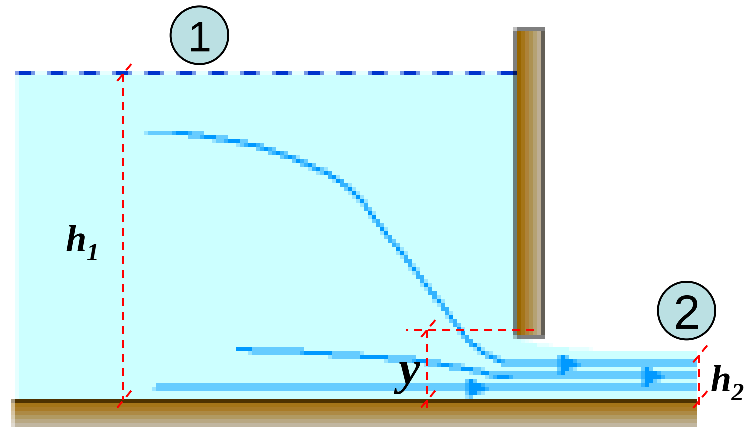
$$\boldsymbol{v^2 = 2g(h_2 - h_1)}$$

$$\boldsymbol{v = \sqrt{2gh}}$$ *Theoretical velocity*

Fluid Flow Measurement

Gates

A *gate* is an opening in a dam or other hydraulic structure to control the passage of water. It has the same hydraulic properties as the orifice.



$$E_1 = E_2$$

$$\frac{v_1^2}{2g} + \overset{h_1}{\cancel{\frac{p_1}{\gamma}}} + \overset{0}{\cancel{z_1}} = \frac{v_2^2}{2g} + \overset{h_2}{\cancel{\frac{p_2}{\gamma}}} + \overset{0}{\cancel{z_2}}$$

$$\frac{v_2^2}{2g} - \frac{v_1^2}{2g} = h_1 - h_2$$

$$v_2^2 - v_1^2 = 2g(h_1 - h_2)$$

$$v_2^2 = 2g(h_1 - h_2) + v_1^2$$

$$\boldsymbol{v_2 = v_T = \sqrt{2g(h_1 - h_2) + v_1^2}}$$

Theoretical velocity

Fluid Flow Measurement

Gates

Actual velocity, $\boldsymbol{v = C_v\sqrt{2g(h_1 - h_2)\ + v_1^2}}$

Actual discharge, $\boldsymbol{Q = CA\sqrt{2g(h_1 - h_2)\ + v_1^2}}$

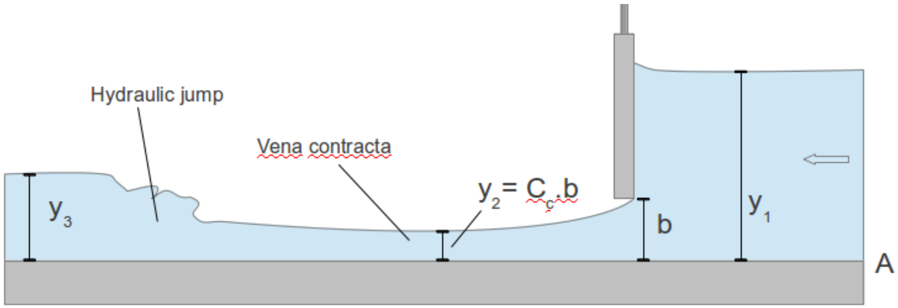
Coefficient of contraction, $\boldsymbol{C_c = \dfrac{h_2}{y}}$

where,

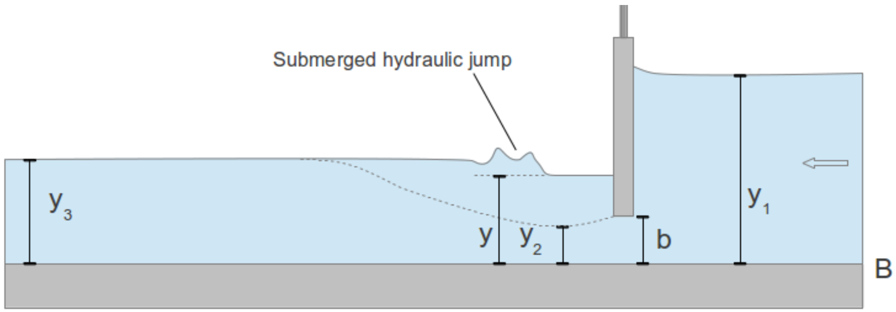
$\boldsymbol{C} = C_c C_v\ (varies\ from\ 0.61\ to\ 0.91)$

$\boldsymbol{A} = by$

$\boldsymbol{b} = width\ of\ the\ flume$



Free flow



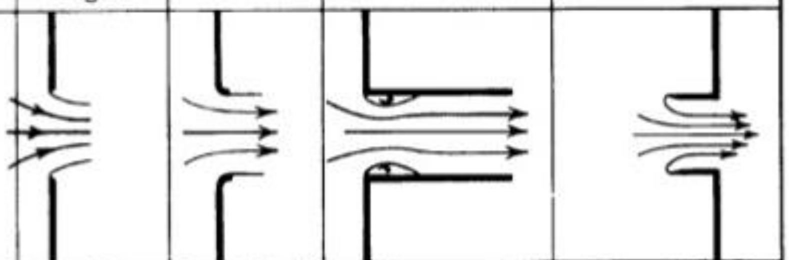
Submerged flow

Tubes

Standard Short Tube and Re-entrant Tube

A *standard short tube* is the one with a square-cornered entrance and has a length of about 2.5 times its internal diameter.

Orifices and their Nominal Coefficients

	Sharp edged	Rounded	Short tube	Borda
				
C	0.61	0.98	0.80	0.51
C_c	0.62	1.00	1.00	0.52
C_v	0.98	0.98	0.80	0.98

Borda's Mouthpiece

A special case of a re-entrant tube, consisting of a thin tube projecting into a tank having a length of about one diameter.

Fluid Flow Measurement

Tubes

Converging Tubes

Conical converging tubes has the form of a frustum of a right circular cone with the larger end adjacent to the tank or reservoir.

Coefficient	Angle of Convergence, Θ								
	0^0	5^0	10^0	15^0	20^0	25^0	30^0	40^0	50^0
C_v	0.829	0.911	0.947	0.965	0.971	0.973	0.976	0.981	0.984
C_c	1.000	0.999	0.992	0.972	0.952	0.935	0.918	0.888	0.859
C	0.829	0.910	0.939	0.938	0.924	0.911	0.896	0.871	0.845

Diverging Tubes

A *diverging tube* has the form of a frustum of a right circular cone with the smaller end adjacent to the reservoir or tank.

Fluid Flow Measurement

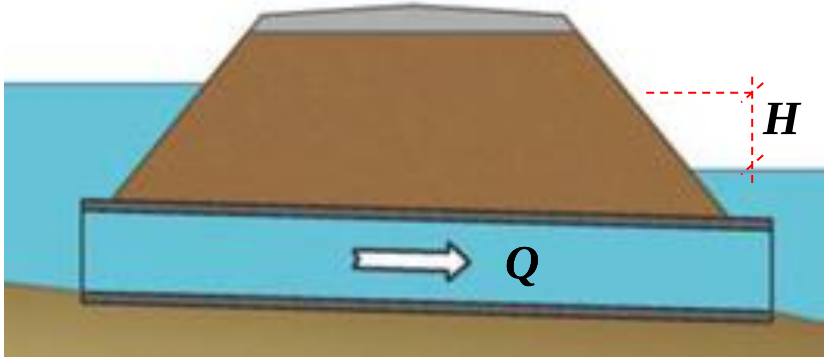
Tubes

Submerged Tubes

An example of submerged tube is a *culvert* conveying water through embankments. The discharge through a submerged tube is given by the formula:

$$Q = CA\sqrt{2gH}$$

where ***C*** is the coefficient of discharge, ***A*** is the area of the opening, and ***H*** is the difference in elevation of the liquid surfaces.

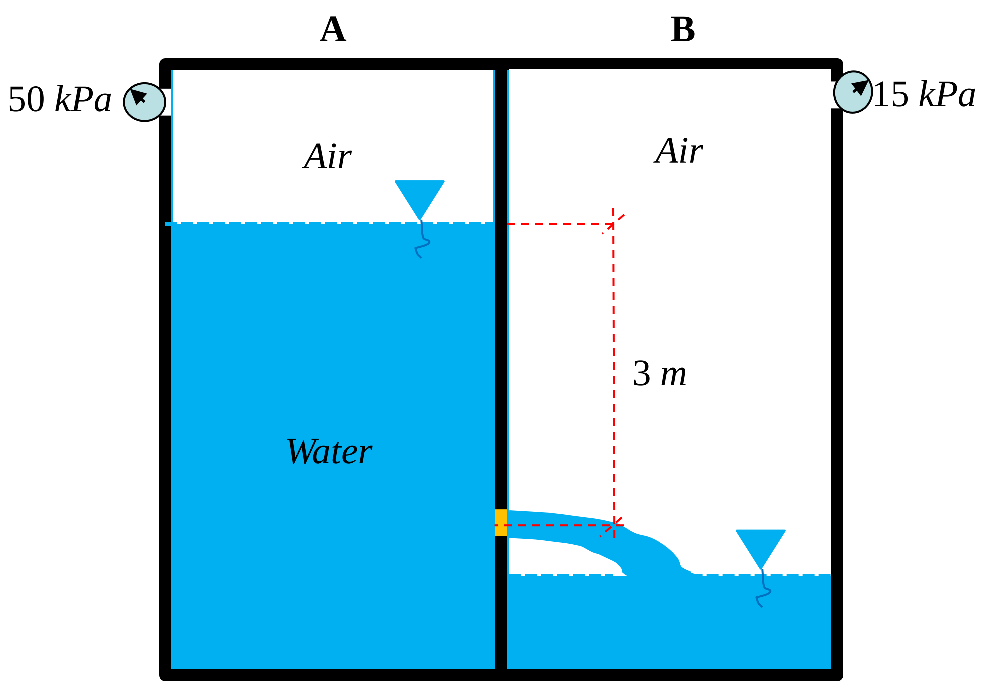


Fundamentals of Fluid Flow

Problem Set 14

Problem 1

Calculate the discharge through the 140-*mm* diameter orifice shown.



Problem Set 14

Problem 1

Solution:

$$Q = CA\sqrt{2gH}$$

$$\boldsymbol{H = H_{upstream} - H_{downstream}}$$

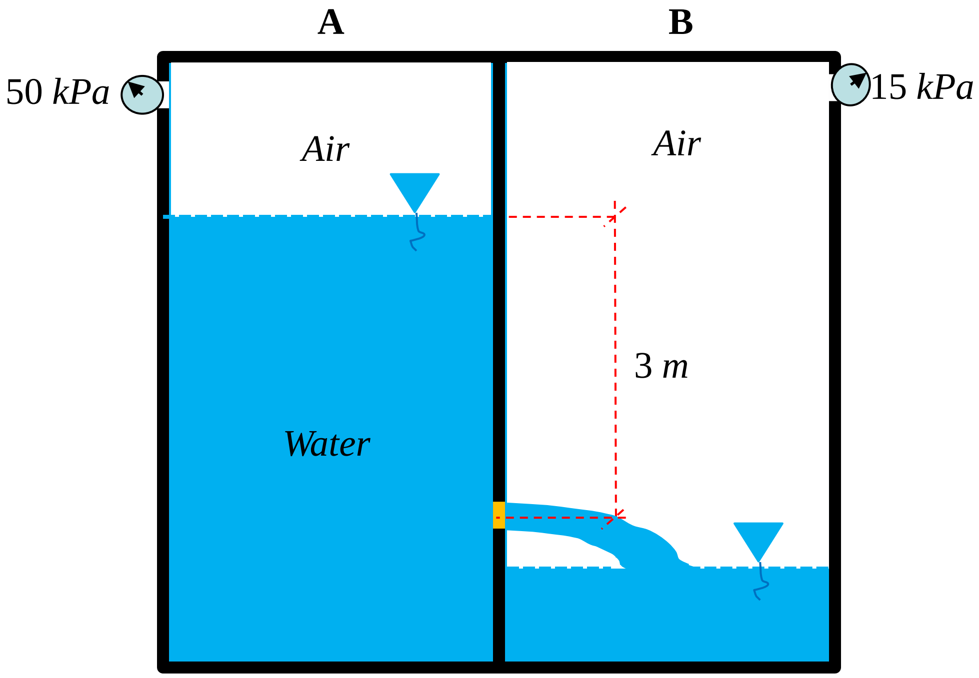
$$H = 3 + \frac{50}{9.81} - \frac{15}{9.81}$$

$$H = 6.568\ m$$

$$Q = 0.62\left[\frac{\pi(0.14)^2}{4}\right]\sqrt{2(9.81)(6.568)}$$

$$\boldsymbol{Q = 0.108\ m^3/s}$$

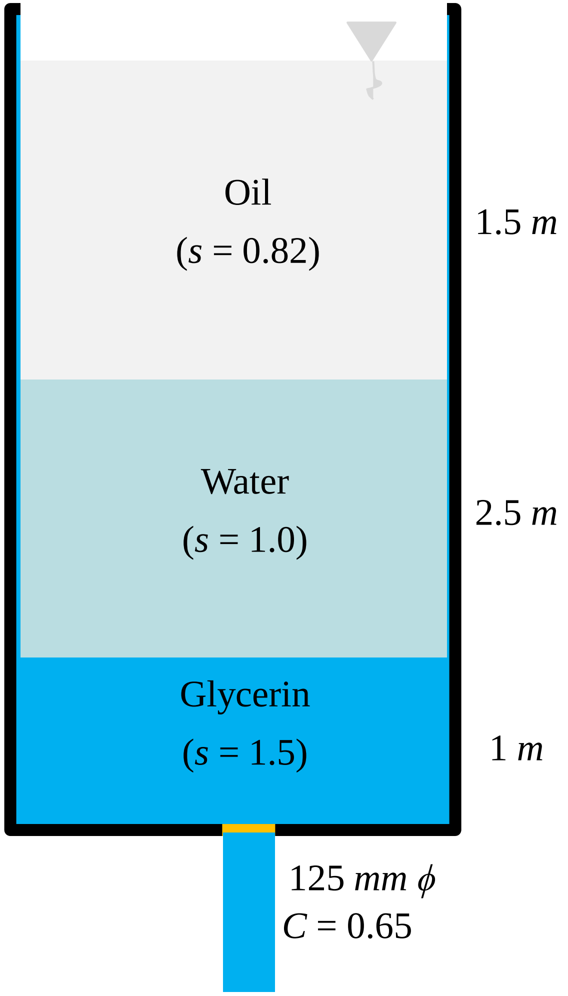
$$\boldsymbol{H = h + \frac{v_A^2}{2g} + \frac{p_A}{\gamma} - \frac{p_B}{\gamma}}$$



Problem Set 14

Problem 2

An open cylindrical tank, 2.4 m in diameter and 6 m tall has 1 m of glycerin (s = 1.5), 2.5 m of water, and 1.5 m of oil (s = 0.82). Determine the discharge through the 125 mm diameter orifice located at the bottom of the tank. Assume C = 0.65.



Fundamentals of Fluid Flow

Problem Set 14

Problem 2

Solution:

$$Q = CA\sqrt{2gH}$$

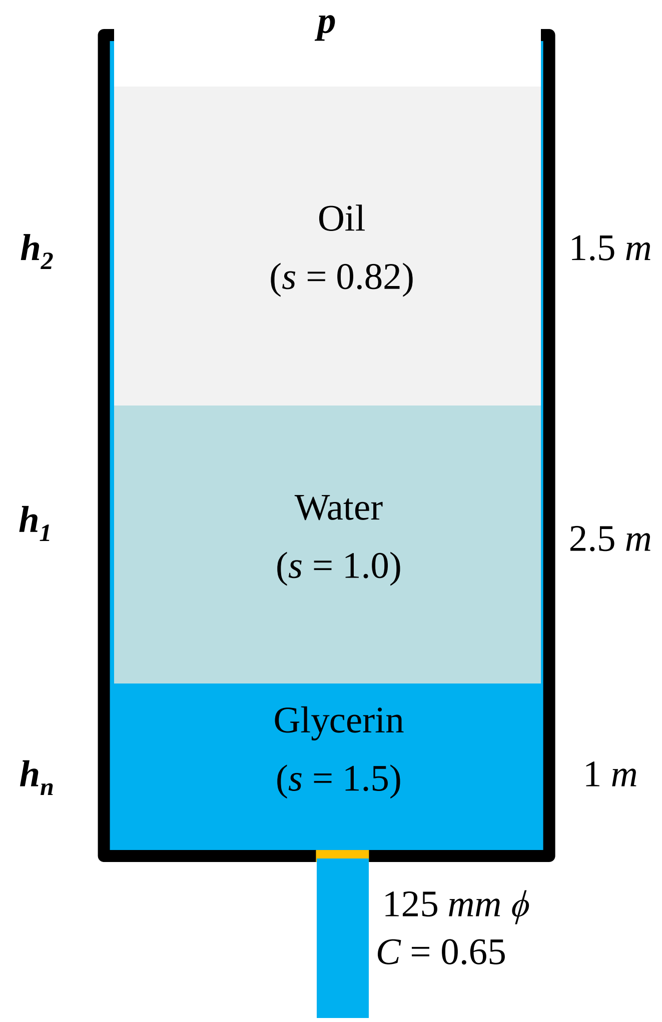
$$\boldsymbol{H = h_n + h_1\left(\frac{s_1}{s_n}\right) + \cdots \frac{p}{\gamma_n}}$$

$$H = 1 + 2.5\left(\frac{1}{1.5}\right) + 1.5\left(\frac{0.82}{1.5}\right)$$

$$H = 3.487\ m\ of\ glycerin$$

$$Q = 0.65\left[\frac{\pi(0.125)^2}{4}\right]\sqrt{2(9.81)(3.487)}$$

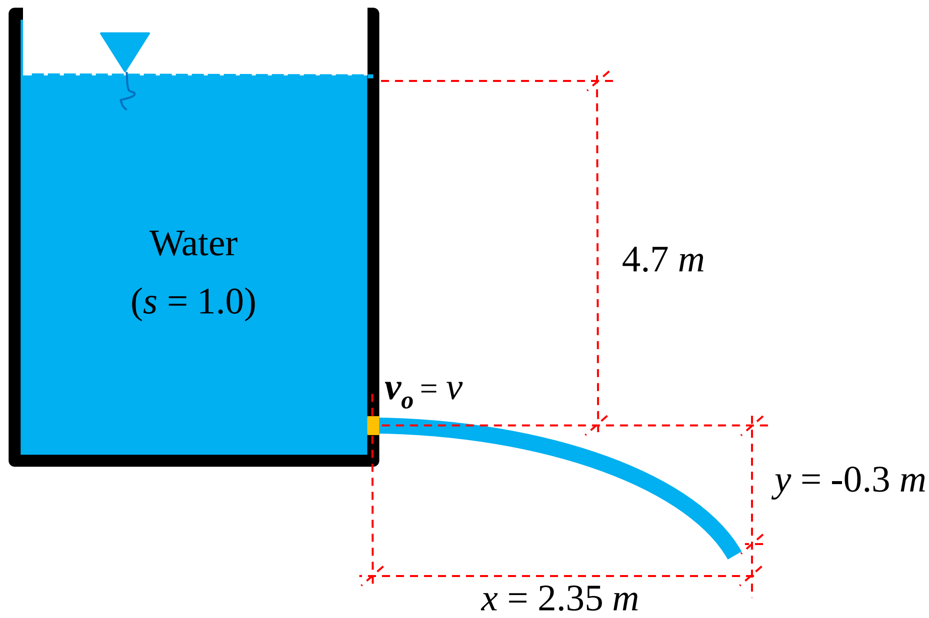
$$\boldsymbol{Q = 0.066\ m^3/s}$$



Problem Set 14

Problem 3

A calibration test of a 12.5 *mm* diameter circular sharp-edged orifice in a vertical side of a large tank showed a discharge of 590 *N* of water in 81 *sec* at a constant head of 4.70 *m*. Measurement of the jet showed that it travelled 2.35 *m* horizontally while dropping 300 *mm*. Compute the three coefficients.



Problem Set 14

Problem 3

Solution:

Theoretical values:

$$v_T = \sqrt{2gH} = \sqrt{2(9.81)(4.7)}$$

$$\boldsymbol{v_T = 9.603\ m/s}$$

$$Q_T = Av_T = \left[\frac{\pi(0.0125)^2}{4}\right](9.603)$$

$$\boldsymbol{Q_T = 0.001178\ m^3/s}$$

Actual values:

$$Q = \frac{Volume}{time}\ (steady\ flow)$$

$$V = \frac{W}{\gamma} = \frac{590}{9810} = 0.0601\ m^3$$

$$Q = \frac{0.0601\ m^3}{81\ s}$$

$$\boldsymbol{Q_T = 0.000743\ m^3/s}$$

Problem Set 14

Problem 3

Solution:

Actual values (velocity):

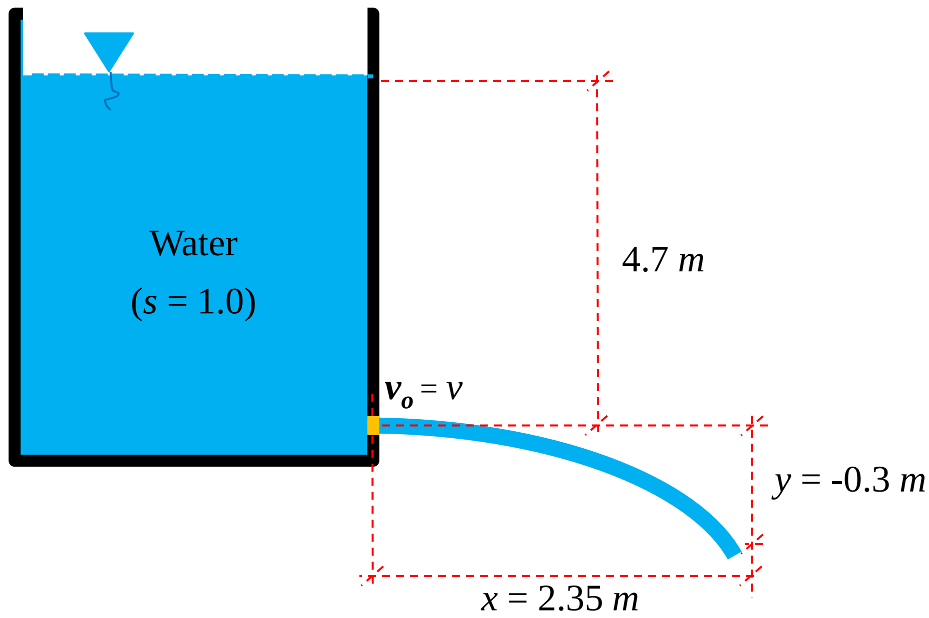
Trajectory of a projectile

The height of y of the projectile at distance x:

$$y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$$

$$-0.3 = 2.35\tan 0^0 - \frac{9.81(2.35)^2}{2v^2\cos^2(0^0)}$$

$$\boldsymbol{v = 9.502\ m/s}$$



Fundamentals of Fluid Flow

Problem Set 14

Problem 3

Solution:

Coefficients:

$$Coefficient\ of\ velocity, \boldsymbol{C_v} = \frac{\boldsymbol{v}}{\boldsymbol{v_T}} = \frac{9.502}{9.603} = \mathbf{0.989}$$

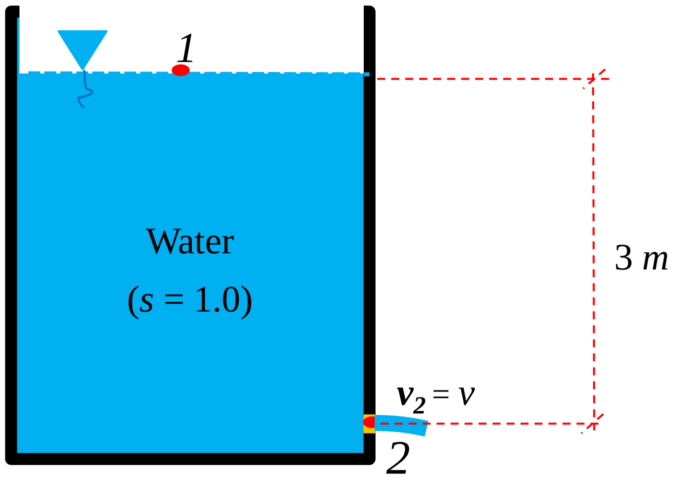
$$Coefficient\ of\ discharge, \boldsymbol{C} = \frac{\boldsymbol{Q}}{\boldsymbol{Q_T}} = \frac{0.000743}{0.001178} = \mathbf{0.631}$$

$$Coefficient\ of\ contraction, \boldsymbol{C_c} = \frac{\boldsymbol{C}}{\boldsymbol{C_v}} = \frac{0.631}{0.989} = \mathbf{0.638}$$

Problem Set 14

Problem 4

A 50-*mm* diameter circular sharp-edged orifice at the side of a tank discharges water under a head of 3 *m*. If the coefficient of contraction C_c = 0.63 and the head loss is 240 *mm*, compute the discharge and the coefficients of velocity C_v and discharge C.



Fundamentals of Fluid Flow

Problem Set 14

Problem 3

Solution:

*Energy equation between **1** and **2** (actual):*

$$E_1 - HL_{1-2} = E_2$$

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - HL = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$(0 + 0 + 3) - 0.24 = \left(\frac{v^2}{2g} + 0 + 0\right)$$

$$\frac{v^2}{2g} = 2.76\ m$$

$v = 7.359\ m/s$ (actual velocity)

$$v_T = \sqrt{2gH} = \sqrt{2(9.81)(3)}$$

$v_T = 7.672\ m/s$ (theoretical velocity)

Coefficient of velocity

$$\boldsymbol{C_v} = \frac{\boldsymbol{v}}{\boldsymbol{v_T}} = \frac{7.359}{7.672} = \mathbf{0.959}$$

Coefficient of discharge

$$\boldsymbol{C} = \boldsymbol{C_c C_v} = 0.63(0.959) = \mathbf{0.604}$$

Problem Set 14

Problem 5

A jet is issued from the side of a tank under a constant head of 3 *m*. The side of the tank has an inclination of *1H* to *1V*. The total depth of water in the tank is 6.70 *m*. Neglecting air resistance and assuming C_v = 1.0, determine the following:

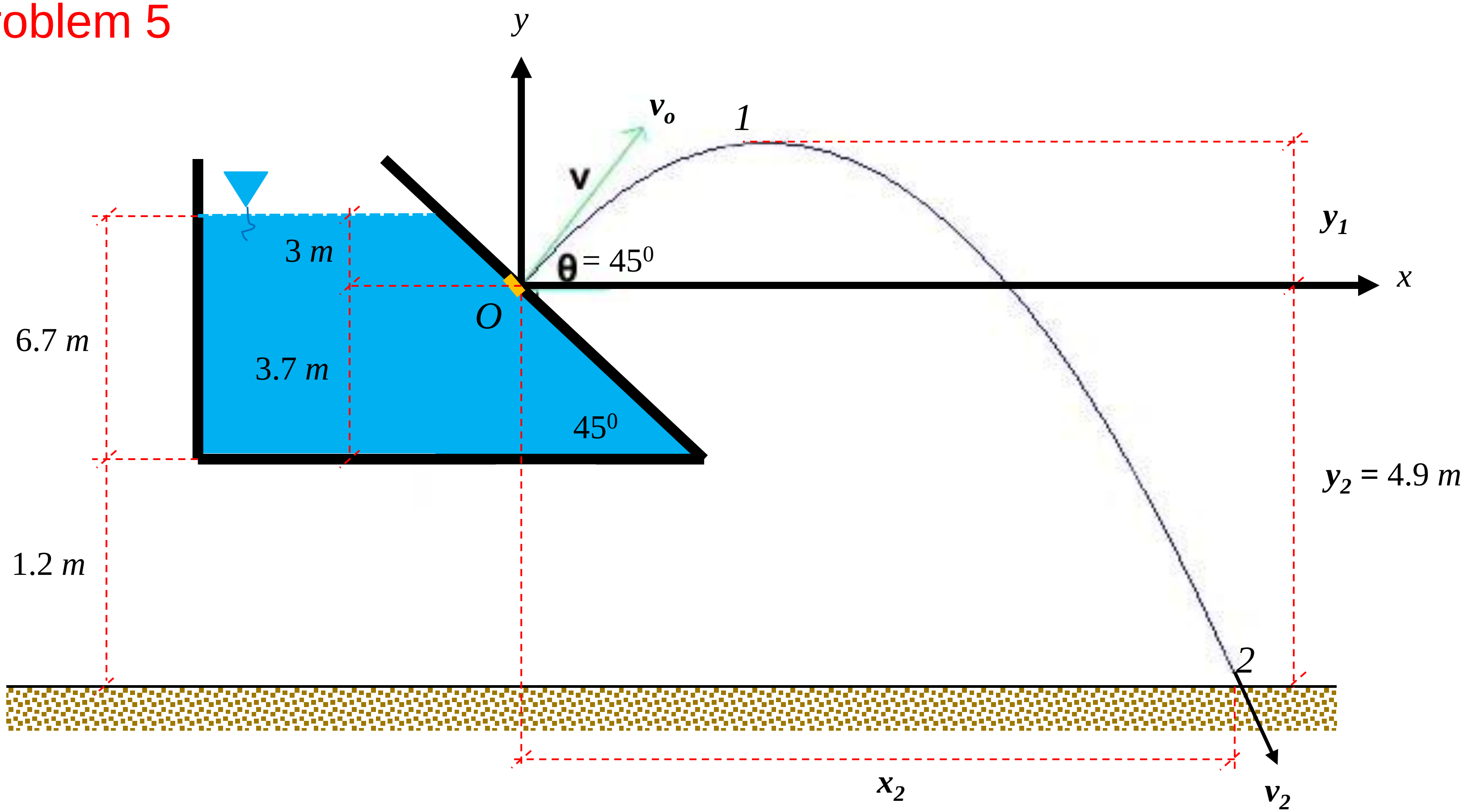
5.1 The maximum height to which the jet will rise.

5.2 The point it strike a horizontal plane 1.20 *m* below the bottom of the tank.

5.3 The velocity of the jet as it strikes the ground.

Problem Set 14

Problem 5



Problem Set 14

Problem 5

Solution:

Actual velocity of jet at the orifice, $\boldsymbol{v_o}$:

$$v_o = C_v v_T = C_v \sqrt{2gH} = 1\sqrt{2(9.81)(3)}$$

$$v_o = 7.672\ m/s$$

5.1 Maximum height (at point 1, $\boldsymbol{v_y} = 0$*)*

From physics, the vertical component of velocity,

$$\boldsymbol{v_y^2 = v_{oy}^2 - 2gy}$$

$$0 = (7.672 sin 45^0) - 2(9.81)(y_1)$$

$$\boldsymbol{y_1 = 1.5\ m}$$

Problem Set 14

Problem 5

Solution:

5.2 Point it strike the ground, $\boldsymbol{x_2}$

(at point 2, $\boldsymbol{y_2}$ *= -4.9 m)*

$$y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$$

$$-4.9 = x_2\tan 45^0 - \frac{9.81(x_2)^2}{2(7.672)^2\cos^2(45^0)}$$

$$\boldsymbol{x_2 = 9.18\ m}$$

5.3 Velocity of the jet as it strikes the ground

Work-Energy equation between ***O*** *and* ***2***

$$KE_O + Wy_2 = KE_2$$

$$\frac{1}{2}\frac{W}{g}v_o^2 + Wh = \frac{1}{2}\frac{W}{g}v_2^2$$

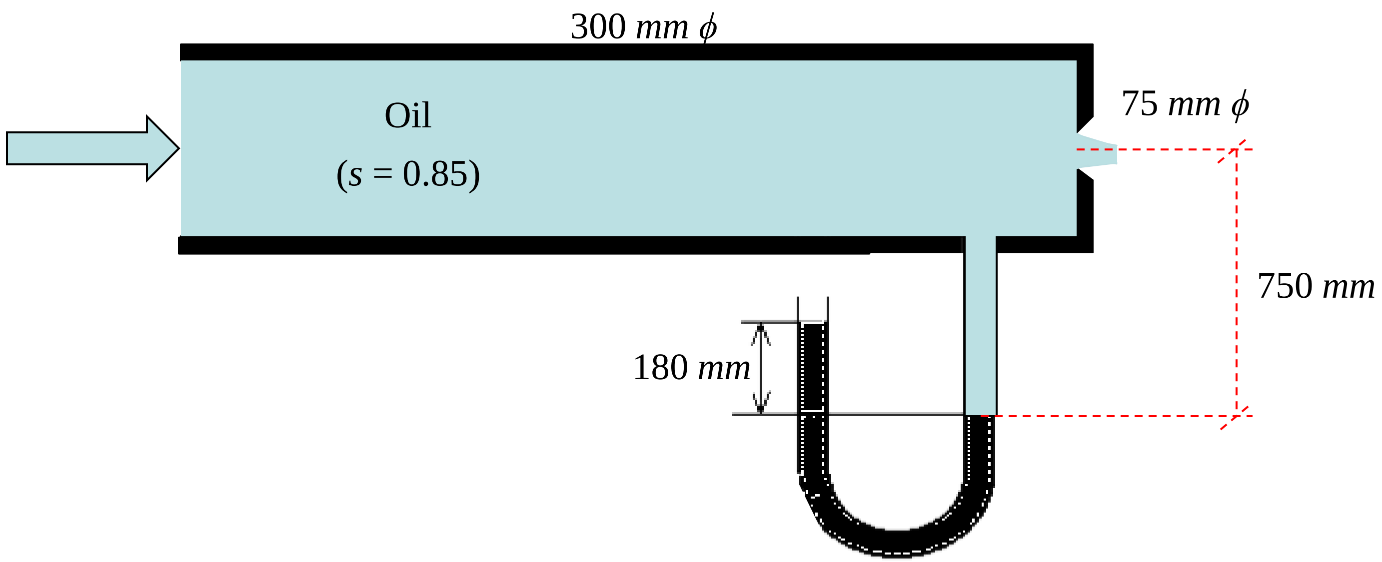
$$\frac{7.672^2}{2(9.81)} + 4.9 = \frac{v_2^2}{2g}$$

$$\boldsymbol{v_2 = 12.45\ m/s}$$

Problem Set 14

Problem 6

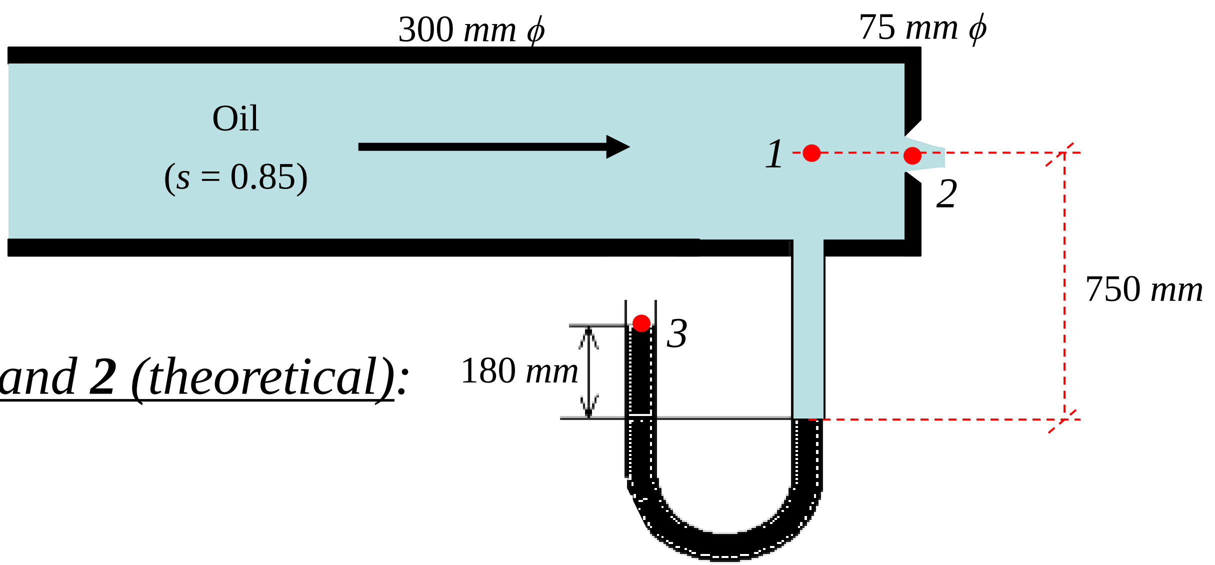
Oil discharges from a pipe through a sharp-crested round orifice as shown in the figure. The coefficients of contraction and velocity are 0.62 and 0.98, respectively. Calculate the discharge through the orifice and the diameter and actual velocity in the jet.



Problem Set 14

Problem 6

Solution:



Discharge $\mathbf{Q} = \mathbf{Q_1} = \mathbf{Q_2}$

*Energy equation between **1** and **2** (theoretical):*

$$E_1 = E_2$$

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

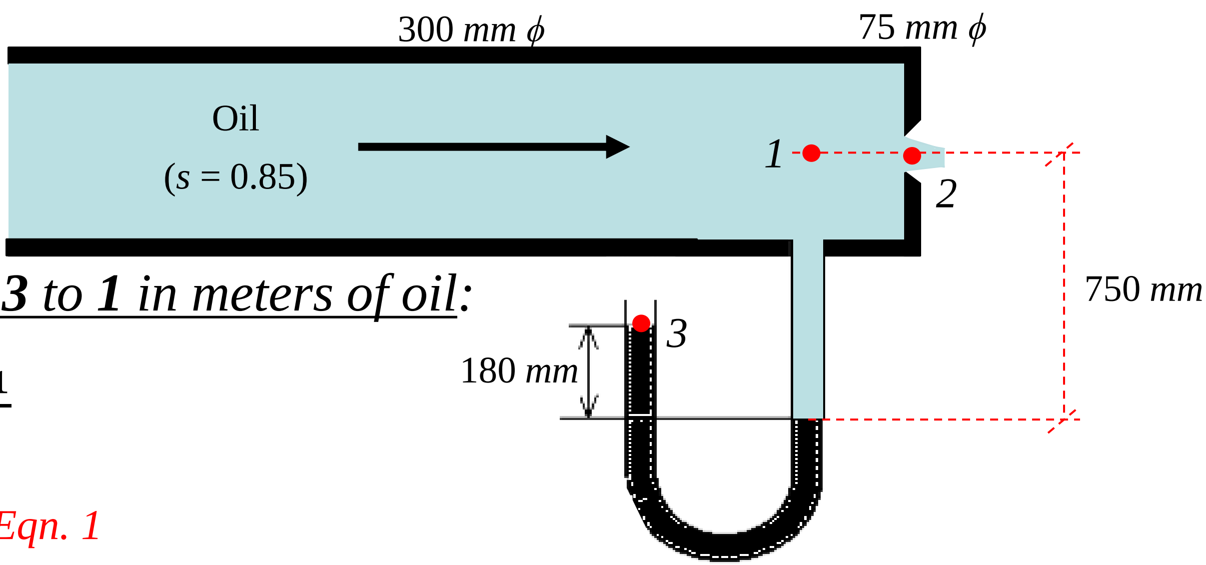
$$\left(\frac{8Q_T^2}{\pi^2 g(0.30)^4} + \frac{p_1}{\gamma} + 0\right) = \left(\frac{8Q_T^2}{\pi^2 g(0.075)^4} + 0 + 0\right)$$

$$\frac{p_1}{\gamma} = 2601Q_T^2 \longrightarrow \textit{Eqn. 1}$$

Problem Set 14

Problem 6

Solution:



*Sum-up pressure head from **3** to **1** in meters of oil:*

$$\frac{p_3}{\gamma} + 0.18\left(\frac{13.6}{0.85}\right) - 0.75 = \frac{p_1}{\gamma}$$

$$\frac{p_1}{\gamma} = 2.13\ m\ of\ oil$$

in Eqn. 1

$$2.13 = 2601 Q_T^2$$

$$Q_T = 0.0286\ m^3/s$$

Actual discharge, $Q = CQ_T = C_c C_v Q_T = 0.62(0.98)(0.0286)$

Actual discharge, $\boldsymbol{Q = 0.0174\ m^3/s}$ or 17.4 *L/s*

Problem Set 14

Problem 6

Solution:

Actual diameter of the jet, $\boldsymbol{d}$:

$$C_c = \frac{a}{A} = \frac{d^2}{D^2}$$

$$0.62 = \frac{d^2}{75^2}$$

$$\boldsymbol{d = 59.1\ mm}$$

Theoretical velocity:

$$v_T = \frac{Q_T}{A} = \frac{0.0286}{\frac{\pi}{4}(0.075)^2} = 6.474\ m/s$$

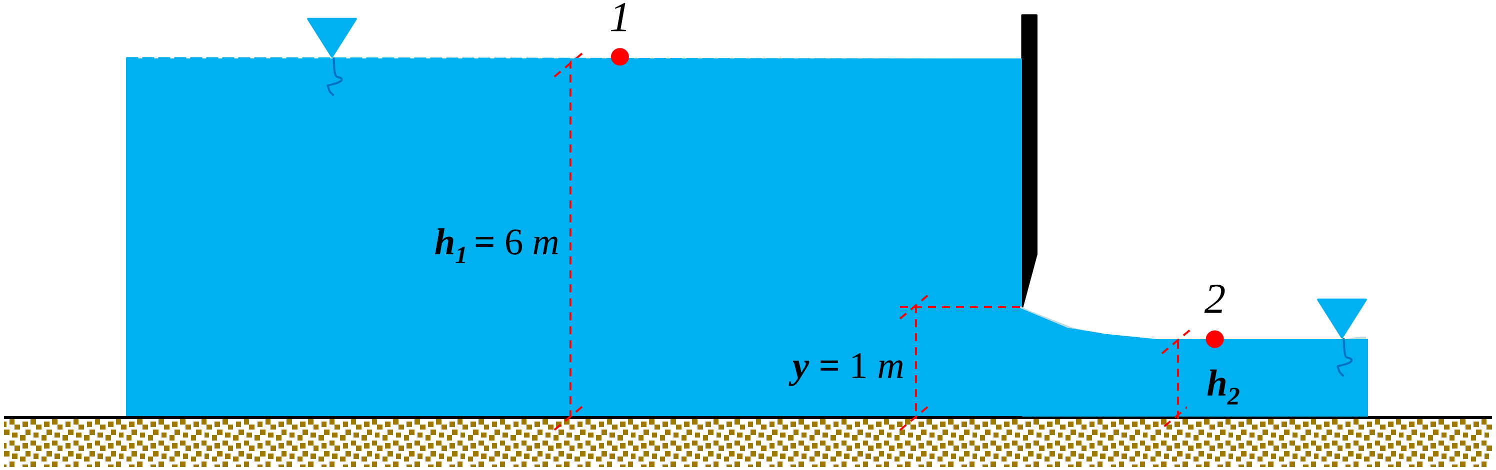
Actual velocity, $v = C_v v_T = 0.98(6.474)$

Actual velocity, $\boldsymbol{v = 6.344\ m/s}$

Problem Set 14

Problem 7

A sluice gate flows into a horizontal channel as shown in the figure. Determine the flow through the gate per meter width when y = 1.0 m and h_1 = 6 m. Assume that the pressure distribution at sections ***1*** and ***2*** to be atmospheric and neglect friction losses in the channel. Use coefficient of contraction C_c = 0.85 and coefficient of velocity C_v = 0.95.



Fundamentals of Fluid Flow

Problem Set 14

Problem 7

Solution:

$h_2 = C_c y = 0.85(1) = 0.85\ m$

*<u>Energy equation between **1** and **2** (theoretical)</u>:*

$E_1 = E_2$

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$\left(\frac{v_1^2}{2g} + 0 + 6\right) = \left(\frac{v_2^2}{2g} + 0 + 0.85\right)$$

$$\left(\frac{v_2^2 - v_1^2}{2g}\right) = 5.15$$

$(v_2^2 - v_1^2) = 101.043 \longrightarrow$ *Eqn. 1*

Problem Set 14

Problem 7

Solution:

$$\boldsymbol{Q_1 = Q_2}$$

$$(6\ x\ 1)v_1 = (0.85\ x\ 1)v_2$$

$$v_1 = 0.1417v_2$$

In Eqn. 1,

$$v_2^2 - (0.1417v_2)^2 = 101.043$$

$$v_{2T} = 10.154\ m/s$$

Actual velocity, $v_{2A} = C_v v_{2T} = 0.95(10.154) = 9.6467\ m/s$

Discharge $= A_2 v_{2A} = (0.85\ x\ 1)(9.6467) = \mathbf{8.2\ m^3/s\ per\ meter}$

Problem Set 14

Problem 8

A 150-*mm* diameter horizontal Venturi meter is installed in a 450-*mm* diameter water main. The deflection of mercury in the differential manometer connected from the inlet to the throat is 350 *mm*.

8.1 Determine the discharge neglecting head loss.

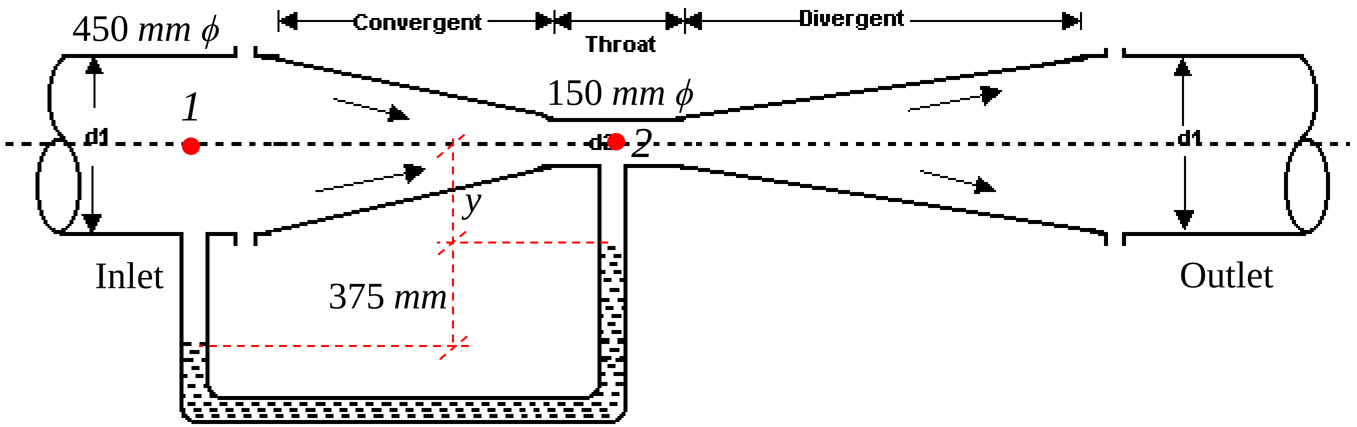
8.2 Compute the discharge if the head loss from the inlet to the throat is 300 *mm* of water.

8.3 What is the meter coefficient?

Fundamentals of Fluid Flow

Problem Set 14

Problem 8



Solution:

$$\boldsymbol{Q_1 = Q_2 = Q}$$

Problem Set 14

Problem 8

Solution:

*<u>Energy equation between **1** and **2** neglecting head loss (theoretical)</u>:*

$$E_1 = E_2$$

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$\left(\frac{8Q_T^2}{\pi^2 g(0.45)^4} + \frac{p_1}{\gamma} + 0\right) = \left(\frac{8Q_T^2}{\pi^2 g(0.15)^4} + \frac{p_2}{\gamma} + 0\right)$$

$$\frac{p_1 - p_2}{\gamma} = 161.2Q_T^2 \longrightarrow \textit{Eqn. 1}$$

Problem Set 14

Problem 8

Solution:

*Sum-up pressure head from **2** to **1** in meters of water:*

$$\frac{p_2}{\gamma} + y + 0.375(13.6) - 0.375 - y = \frac{p_1}{\gamma}$$

$$\frac{p_1 - p_2}{\gamma} = 4.725\, m\ of\ water$$

In Eqn. 1,

$$161.2Q_T^2 = 4.725\, m\ of\ water$$

$$\boldsymbol{Q_T = 0.1712\, m^3/s}$$

Problem Set 14

Problem 8

Solution:

*Energy equation between **1** and **2** considering head loss (actual):*

$$E_1 - HL_{1-2} = E_2$$

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - HL = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$\left(\frac{8Q^2}{\pi^2 g(0.45)^4} + \frac{p_1}{\gamma} + 0\right) - 0.30 = \left(\frac{8Q^2}{\pi^2 g(0.15)^4} + \frac{p_2}{\gamma} + 0\right)$$

$$\frac{p_1 - p_2}{\gamma} - 0.30 = 161.2Q^2 \longrightarrow \textit{Eqn. 2}$$

$$4.725 - 0.30 = 161.2Q^2$$

$$\boldsymbol{Q = 0.1657\ m^3/s}$$

$$Meter\ coefficient$$

$$\boldsymbol{C} = \frac{\boldsymbol{Q}}{\boldsymbol{Q_T}} = \frac{0.1657}{0.1712} = \boldsymbol{0.968}$$

Fundamentals of Fluid Flow

Problem Set 14

Problem 9

A vertical Venturi meter, 150-*mm* in diameter is connected to a 300-*mm* diameter pipe. The vertical distance from the inlet to the throat being 750 *mm*. If the deflection of mercury in the differential manometer connected from the inlet to the throat is 360 *mm*, determine the flow of water through the meter if the meter coefficient is 0.68. Determine also the head loss from the inlet to the throat.